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Intersecting M-branes as Four-Dimensional Black Holes

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Abstract

We present two 1/8 supersymmetric intersecting p-brane solutions of 11-dimensional supergravity which upon compactification to four dimensions reduce to extremal dyonic black holes with finite area of horizon. The first solution is a configuration of three intersecting 5-branes with an extra momentum flow along the common string. The second describes a system of two 2-branes and two 5-branes. Related (by compactification and T-duality) solution of type IIB theory corresponds to a completely symmetric configuration of four intersecting 3-branes. We suggest methods for counting the BPS degeneracy of three intersecting 5-branes which, in the macroscopic limit, reproduce the Bekenstein-Hawking entropy.

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1. Introduction

The existence of supersymmetric extremal dyonic black holes with finite area of the horizon provides a possibility of a statistical understanding [1] of the Bekenstein-Hawking entropy from the point of view of string theory [2,3,4]. Such black hole solutions are found in four [5,6,7] and five [4,8] dimensions but not in $D > 5$ [9,10]. While the D-brane BPS state counting derivation of the entropy is relatively straightforward for the $D = 5$ black holes [4,11], it is less transparent in the $D = 4$ case, a complication being the presence of a solitonic 5-brane or Kaluza-Klein monopole in addition to a D-brane configuration in the descriptions used in [12,13].

One may hope to find a different lifting of the dyonic $D = 4$ black hole to $D = 10$ string theory that may correspond to a purely D-brane configuration. A related question is about the embedding of the $D = 4$ dyonic black holes into $D = 11$ supergravity (M-theory) which would allow to reproduce their entropy by counting the corresponding BPS states using the M-brane approach similar to the one applied in the $D = 5$ black hole case in [14].

As was found in [15], the (three-charge, finite area) $D = 5$ extremal black hole can be represented in M-theory by a configuration of orthogonally intersecting 2-brane and 5-brane (i.e. $2\perp5$) with a momentum flow along the common string, or by configuration of three 2-branes intersecting over a point ($2\perp2\perp2$). A particular embedding of (four-charge, finite area) $D = 4$ black hole into $D = 11$ theory given in [15] can be interpreted as a similar $2\perp5$ configuration 'superposed' with a Kaluza-Klein monopole.

Below we shall demonstrate that it is possible to get rid of the complication associated with having the Kaluza-Klein monopole. There exists a simple 1/8 supersymmetric configuration of *four* intersecting M-branes ($2\perp2\perp5\perp5$) with diagonal $D = 11$ metric. Upon compactification along six isometric directions it reduces to the dyonic $D = 4$ black hole with finite area and all scalars being regular at the horizon.

The corresponding $2\perp2\perp4\perp4$ solution of type IIA $D = 10$ superstring theory (obtained by dimensional reduction along a direction common to the two 5-branes) is T-dual to a $D = 10$ solution of type IIB theory which describes a remarkably symmetric configuration of *four* intersecting 3-branes.[1]

[1] Similar D-brane configuration was discussed in [16,17]. Note that it is a combination of *four* and not *three* intersecting 3-branes that is related (for the special choice of equal charges) to the non-dilatonic ($a = 0$) RN $D = 4$ black hole. T-dual configuration of one 0-brane and three intersecting 4-branes of type IIA theory was considered in [18].

Our discussion will follow closely that of [15] where an approach to constructing intersecting supersymmetric p-brane solutions (generalising that of [19]) was presented.[2] The supersymmetric configurations of two or three intersecting 2- and 5-branes of $D = 11$ supergravity which preserve 1/4 or 1/8 of maximal supersymmetry are $2\perp 2$, $5\perp 5$, $2\perp 5$, $2\perp 2\perp 2$, $5\perp 5\perp 5$, $2\perp 2\perp 5$ and $2\perp 5\perp 5$. Two 2-branes can intersect over a point, two 5-branes – over a 3-brane (which in turn can intersect over a string), 2-brane and 5-brane can intersect over a string [19]. There exists a simple 'harmonic function' rule which governs the construction of composite supersymmetric p-brane solutions in both $D = 10$ and $D = 11$: a separate harmonic function is assigned to each constituent 1/2 supersymmetric p-brane.

Most of the configurations with *four* intersecting M-branes, namely, $2\perp 2\perp 2\perp 2$, $2\perp 2\perp 2\perp 5$ and $5\perp 5\perp 5\perp 2$ are 1/16 supersymmetric and have transverse x-space dimension equal to two ($5\perp 5\perp 5\perp 5$ configuration with 5-branes intersecting over 3-branes to preserve supersymmetry does not fit into 11-dimensional space-time). Being described in terms of harmonic functions of x they are thus not asymptotically flat in transverse directions. There exists, however, a remarkable exception – the configuration $2\perp 2\perp 5\perp 5$ which (like $5\perp 2\perp 2$, $5\perp 5\perp 2$ and $5\perp 5\perp 5$) has transverse dimension equal to three and the fraction of unbroken supersymmetry equal to 1/8 (Section 3). Upon compactification to $D = 4$ it reduces to the extremal dyonic black hole with *four* different charges and finite area of the horizon.

Similar $D = 4$ black hole background can be obtained also from the the 'boosted' version of the $D = 11$ $5\perp 5\perp 5$ solution [15] (Section 2)[3] as well from the $3\perp 3\perp 3\perp 3$ solution of $D = 10$ type IIB theory (Section 4). The two $D = 11$ configurations $5\perp 5\perp 5$+'boost' and $2\perp 2\perp 5\perp 5$ reduce in $D = 10$ to $0\perp 4\perp 4\perp 4$ and $2\perp 2\perp 4\perp 4$ solutions of $D = 10$ type IIA theory which are related by T-duality.

In Section 5 we shall suggest methods for counting the BPS entropy of three intersecting 5-branes which reproduce the Bekenstein-Hawking entropy of the $D = 4$ black hole. This seems to explain the microscopic origin of the entropy directly in 11-dimensional terms.

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[2] Intersecting p-brane solutions in [19,15] and below are isometric in all directions internal to all constituent p-branes (the background fields depend only on the remaining common transverse directions). They are different from possible virtual configurations where, e.g., a (p-2)-brane ends (in transverse radial direction) on a p-brane [20]. A configuration of p-brane and p′-brane intersecting in p+p′-space may be also considered as a special anisotropic p+p′-brane. There may exist more general solutions (with constituent p-branes effectively having different transverse spaces [19,21]) which may 'interpolate' between intersecting p-brane solutions and solutions with one p-brane ending on another in the transverse direction of the latter.

[3] The 'boost' along the common string corresponds to a Kaluza-Klein electric charge part in the $D = 11$ metric which is 'dual' to a Kaluza-Klein monopole part present in the $D = 11$ embedding of dyonic black hole in [15].

2. 'Boosted' $5\bot5\bot5$ solution of $D = 11$ theory

The $D = 11$ background corresponding to $5\bot5\bot5$ configuration [19] is [15]

$$ds^2_{11} = (F_1F_2F_3)^{-2/3}\big[F_1F_2F_3(-dt^2 + dy_1^2) \tag{2.1}$$

$$+ F_2F_3(dy_2^2 + dy_3^2) + F_1F_3(dy_4^2 + dy_5^2) + F_1F_2(dy_6^2 + dy_7^2) + dx_sdx_s\big] ,$$

$$\mathcal{F}_4 = 3(*dF_1^{-1} \wedge dy_2 \wedge dy_3 + *dF_2^{-1} \wedge dy_4 \wedge dy_5 + *dF_3^{-1} \wedge dy_6 \wedge dy_7) . \tag{2.2}$$

Here $\mathcal{F}_4$ is the 4-form field strength and F_i are the inverse powers of harmonic functions of x_s $(s = 1, 2, 3)$. In the simplest 1-center case discussed below $F_i^{-1} = 1 + P_i/r$ $(r^2 = x_sx_s)$. The $*$-duality is defined with respect to the transverse 3-space. The coordinates y_n internal to the three 5-branes can be identified according to the F_i factors inside the square brackets in the metric: $(y_1, y_4, y_5, y_6, y_7)$ belong to the first 5-brane, $(y_1, y_2, y_3, y_6, y_7)$ to the second and $(y_1, y_2, y_3, y_4, y_5)$ to the third. 5-branes intersect over three 3-branes which in turn intersect over a common string along y_1. If $F_2 = F_3 = 1$ the above background reduces to the single 5-brane solution [22] with the harmonic function $H = F_1^{-1}$ independent of the two of transverse coordinates (here y_2, y_3). The case of $F_3 = 1$ describes two 5-branes intersecting over a 3-brane.[4] The special case of $F_1 = F_2 = F_3$ is the solution found in [19].

Compactifying $y_1, .., y_7$ on circles we learn that the effective 'radii' (scalar moduli fields in $D = 4$) behave regularly both at $r = \infty$ and at $r = 0$ with the exception of the 'radius' of y_1. It is possible to stabilize the corresponding scalar by adding a 'boost' along the common string. The metric of the resulting more general solution [15] is (the expression for $\mathcal{F}_4$ remains the same)

$$ds^2_{11} = (F_1F_2F_3)^{-2/3}\big[F_1F_2F_3(dudv + Kdu^2) \tag{2.3}$$

$$+ F_2F_3(dy_2^2 + dy_3^2) + F_1F_3(dy_4^2 + dy_5^2) + F_1F_2(dy_6^2 + dy_7^2) + dx_sdx_s\big] .$$

Here $u = y_1 - t,\ v = 2t$ and K is a harmonic function of the three coordinates x_s. A non-trivial $K = 1 + Q/r$ describes a momentum flow along the string direction.[5]

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[4] The corresponding 1/4 supersymmetric background also has 3-dimensional transverse space and reduces to a $D = 4$ black hole with two charges (it has $a = 1$ black hole metric when two charges are equal). The $5\bot5$ configuration compactified to $D = 10$ gives $4\bot4$ solution of type IIA theory which is T-dual to $3\bot3$ solution of type IIB theory.

[5] The metric (2.3) with $F_i = 1$ (i.e. $ds^2 = -K^{-1}dt^2 + K[dy_1 + (K^{-1} - 1)dt]^2 + dy_ndy_n + dx_sdx_s$) reduces upon compactification along y_1 direction to the $D = 10$ type IIA R-R 0-brane background [23] with Q playing the role of the KK electric charge.

Q also has an interpretation of The $D=11$ metric (2.3) is regular at the $r=0$ horizon and has a *non-zero* 9-area of the horizon (we assume that all y_n have period L)

$$A_9 = 4\pi L^7 [r^2 K^{1/2} (F_1 F_2 F_3)^{-1/2}]_{r\to 0} = 4\pi L^7 \sqrt{QP_1P_2P_3}\ . \tag{2.4}$$

Compactification along $y_2, ...y_7$ leads to a solitonic $D=5$ string. Remarkably, the corresponding 6-volume is constant so that one gets directly the Einstein-frame metric

$$ds_5^2 = H^{-1}(dudv + Kdu^2) + H^2 dx_s dx_s\ , \qquad H \equiv (F_1F_2F_3)^{-1/3}\ . \tag{2.5}$$

Further compactification along y_1 or u gives the $D=4$ (Einstein-frame) metric which is isomorphic to the one of the dyonic black hole [6]

$$ds_4^2 = -\lambda(r)dt^2 + \lambda^{-1}(r)(dr^2 + r^2 d\Omega_2^2)\ , \tag{2.6}$$

$$\lambda(r) = \sqrt{K^{-1}F_1F_2F_3} = \frac{r^2}{\sqrt{(r+Q)(r+P_1)(r+P_2)(r+P_3)}}\ . \tag{2.7}$$

Note, however, that in contrast to the dyonic black hole background of [6] which has two electric and two magnetic charges here there is one electric (Kaluza-Klein) and 3 magnetic charges. From the $D=4$ point of view the two backgrounds are related by U-duality. The corresponding 2-area of the $r=0$ horizon is of course A_9/L^7.

In the special case when all 4 harmonic functions are equal ($K = F_i = H^{-1}$) the metric (2.3) becomes

$$ds_{11}^2 = H^{-1}dudv + du^2 + dy_2^2 + ... + dy_6^2 + H^2 dx_s dx_s \tag{2.8}$$

$$= \ -H^{-2}dt^2 + H^2 dx_s dx_s + [dy_1 + (H^{-1}-1)dt]^2 + dy_2^2 + ... + dy_6^2\ ,$$

and corresponds to a charged solitonic string in $D=5$ or the Reissner-Nordström ($a=0$) black hole in $D=4$ ('unboosted' $5\perp5\perp5$ solution with $K=1$ and equal F_i reduces to $a=1/\sqrt{3}$ dilatonic $D=4$ black hole [19]).

A compactification of this $5\perp5\perp5$+'boost' configuration to $D=10$ along y_1 gives a type IIA solution corresponding to three 4-branes intersecting over 2-branes plus additional Kaluza-Klein (Ramond-Ramond vector) electric charge background, or, equivalently, to the $0\perp4\perp4\perp4$ configuration of three 4-branes intersecting over 2-branes which in turn intersect over a 0-brane. If instead we compactify along a direction common only to two of the three 5-branes we get $4\perp4\perp5$+'boost' type IIA solution.[6] Other related solutions of type IIA and IIB theories can be obtained by applying T-duality and $SL(2,Z)$ duality.

[6] This may be compared to another type IIA configuration (consisting of solitonic 5-brane lying within a R-R 6-brane, both being intersected over a 'boosted' string by a R-R 2-brane) which also reduces [12,15] to the dyonic $D=4$ black hole.

3. $2\perp2\perp5\perp5$ solution of $D = 11$ theory

Solutions with four intersecting M-branes are constructed according to the rules discussed in [15]. The $2\perp2\perp5\perp5$ configuration is described by the following background

$$ds_{11}^2 = (T_1T_2)^{-1/3}(F_1F_2)^{-2/3}\big[- T_1T_2F_1F_2\ dt^2 \tag{3.1}$$
$$+ T_1F_1dy_1^2 + T_1F_2dy_2^2 + T_2F_1dy_3^2 + T_2F_2dy_4^2 + F_1F_2(dy_5^2 + dy_6^2 + dy_7^2) + dx_sdx_s\big] \ ,$$

$$\mathcal{F}_4 = -3dt \wedge (dT_1 \wedge dy_1 \wedge dy_2 + dT_2 \wedge dy_3 \wedge dy_4) \tag{3.2}$$
$$+ 3(*dF_1^{-1} \wedge dy_2 \wedge dy_4 + *dF_2^{-1} \wedge dy_1 \wedge dy_3) \ .$$

Here T_i^{-1} are harmonic functions corresponding to the 2-branes and F_i^{-1} are harmonic functions corresponding to the 5-branes, i.e.

$$T_i^{-1} = 1 + \frac{Q_i}{r} \ , \qquad F_i^{-1} = 1 + \frac{P_i}{r} \ . \tag{3.3}$$

(y_1, y_2) belong to the first and (y_3, y_4) to the second 2-brane. $(y_1, y_3, y_5, y_6, y_7)$ and $(y_2, y_4, y_5, y_6, y_7)$ are the coordinates of the two 5-branes. Each 2-brane intersects each 5-brane over a string. 2-branes intersect over a 0-brane ($x = 0$) and 5-branes intersect over a 3-brane.

Various special cases include, in particular, the 2-brane solution [24] ($T_2 = F_1 = F_2 = 1$), as well as $5\perp5$ ($T_1 = T_2 = 1$) [19] and $2\perp5$ ($T_1 = F_2 = 1$), $2\perp2\perp5$ ($F_2 = 1$), $2\perp5\perp5$ ($T_2 = 1$) [15] configurations (more precisely, their limits when the harmonic functions do not depend on a number of transverse coordinates).

As in the case of the $5\perp5\perp5$+'boost' solution (2.3),(2.2), the metric (3.1) is regular at the $r = 0$ horizon (in particular, all internal y_n-components smoothly interpolate between finite values at $r \to \infty$ and $r \to 0$) with the 9-area of the horizon being (cf.(2.4))

$$A_9 = 4\pi L^7[r^2(T_1T_2F_1F_2)^{-1/2}]_{r\to0} = 4\pi L^7\sqrt{Q_1Q_2P_1P_2} \ . \tag{3.4}$$

The compactification of y_n on 7-torus leads to a $D = 4$ background with the metric which is again the dyonic black hole one (2.6), now with

$$\lambda(r) = \sqrt{T_1T_2F_1F_2} = \frac{r^2}{\sqrt{(r + Q_1)(r + Q_2)(r + P_1)(r + P_2)}} \ . \tag{3.5}$$

In addition, there are two electric and two magnetic vector fields (as in [6]) and also 7 scalar fields. The two electric and two magnetic charges are *directly* related to the 2-brane and 5-brane charges (cf. (3.2)).

When all 4 harmonic functions are equal ($T_i^{-1} = F_i^{-1} = H$) the metric (3.1) becomes (cf. (2.8))

$$ds_{11}^2 = -H^{-2}dt^2 + H^2dx_sdx_s + dy_1^2 + ... + dy_7^2 \ , \tag{3.6}$$

i.e. describes a direct product of a $D = 4$ Reissner-Nordström black hole and a 7-torus.

Thus there exists an embedding of the dyonic $D = 4$ black holes into $D = 11$ theory which corresponds to a remarkably symmetric combination of M-branes only. In contrast to the embeddings with a Kaluza-Klein monopole [15] or electric charge ('boost') (2.3),(2.8) it has a diagonal $D = 11$ metric.

4. $3\perp3\perp3\perp3$ solution of type IIB theory

Dimensional reduction of the background (3.1),(3.2) to $D=10$ along a direction common to the two 5-brane (e.g. y_7) gives a type IIA theory solution representing the R-R p-brane configuration $2\perp2\perp4\perp4$. This configuration is T-dual to $0\perp4\perp4\perp4$ one which is the dimensional reduction of the $5\perp5\perp5+$'boost' solution. This suggests also a relation between the two $D=11$ configurations discussed in Sections 2 and 3.

T-duality along one of the two directions common to 4-branes transforms $2\perp2\perp4\perp4$ into the $3\perp3\perp3\perp3$ solution of type IIB theory. The explicit form of the latter can be found also directly in $D=10$ type IIB theory (i.e. independently of the above $D=11$ construction) using the method of [15], where the 1/4 supersymmetric solution corresponding to two intersecting 3-branes was given. One finds the following $D=10$ metric and self-dual 5-form (other $D=10$ fields are trivial)

$$ds^2_{10} = (T_1T_2T_3T_4)^{-1/2}\big[-T_1T_2T_3T_4\ dt^2 \tag{4.1}$$

$$+ T_1T_2dy_1^2 + T_1T_3dy_2^2 + T_1T_4dy_3^2 + T_2T_3dy_4^2 + T_2T_4dy_5^2 + T_3T_4dy_6^2 + dx_sdx_s\big]\ ,$$

$$\mathcal{F}_5 = dt\wedge(dT_1\wedge dy_1\wedge dy_2\wedge dy_3 + dT_2\wedge dy_1\wedge dy_4\wedge dy_5 \tag{4.2}$$

$$+ dT_3\wedge dy_2\wedge dy_4\wedge dy_6 + dT_4\wedge dy_3\wedge dy_5\wedge dy_6)$$

$$+ *dT_1^{-1}\wedge dy_4\wedge dy_5\wedge dy_6 + *dT_2^{-1}\wedge dy_2\wedge dy_3\wedge dy_6$$

$$+ *dT_3^{-1}\wedge dy_1\wedge dy_3\wedge dy_5 + *dT_4^{-1}\wedge dy_1\wedge dy_2\wedge dy_4\ .$$

The coordinates of the four 3-branes are (y_1,y_2,y_3), (y_1,y_4,y_5), (y_2,y_4,y_6) and (y_3,y_5,y_6), i.e. each pair of 3-branes intersect over a string and all 6 strings intersect at one point. T_i are the inverse harmonic functions corresponding to each 3-brane, $T_i^{-1}=1+Q_i/r$. Like the $2\perp2\perp5\perp5$ background of $D=11$ theory this $D=10$ solution is 1/8 supersymmetric, has 3-dimensional transverse space and diagonal $D=10$ metric.

Its special cases include the single 3-brane [23,25] with harmonic function independent of 3 of 6 transverse coordinates ($T_2=T_3=T_4=1$), $3\perp3$ solution found in [15] ($T_3=T_4=1$) and also $3\perp3\perp3$ configuration ($T_4=1$). The 1/8 supersymmetric $3\perp3\perp3$ configuration also has 3-dimensional transverse space[7] but the corresponding $D=10$ metric

$$ds^2_{10} = (T_1T_2T_3)^{-1/2}\big[-T_1T_2T_3\ dt^2 \tag{4.3}$$

$$+ T_1T_2dy_1^2 + T_1T_3dy_2^2 + T_1dy_3^2 + T_2T_3dy_4^2 + T_2dy_5^2 + T_3dy_6^2 + dx_sdx_s\big]\ ,$$

[7] Similar configurations of three and four intersecting 3-branes, and, in particular, their invariance under the 1/8 fraction of maximal supersymmetry were discussed in D-brane representation in [17,16].

is singular at $r=0$ and has zero area of the $r=0$ horizon.[8]

As in the two $D=11$ cases discussed in the previous sections, the metric of the $3\perp3\perp3\perp3$ solution (4.1) has $r=0$ as a regular horizon with finite 8-area (cf.(2.4),(3.4))

$$A_8 = 4\pi L^6 [r^2 (T_1T_2T_1T_2)^{-1/2}]_{r\to 0} = 4\pi L^6 \sqrt{Q_1Q_2Q_3Q_4}\ . \tag{4.4}$$

A_8/L^6 is the area of the horizon of the corresponding dyonic $D=4$ black hole with the metric (2.6) and

$$\lambda(r) = \sqrt{T_1T_2T_3T_4} = \frac{r^2}{\sqrt{(r+Q_1)(r+Q_2)(r+Q_3)(r+Q_4)}}\ . \tag{4.5}$$

The gauge field configuration here involves 4 pairs of equal electric and magnetic charges. When all charges are equal, the $3\perp3\perp3\perp3$ metric (4.1) compactified to $D=4$ reduces to the $a=0$ black hole metric (while the $3\perp3\perp3$ metric (4.3) reduces to the $a=1/\sqrt{3}$ black hole metric [26]).

5. Entropy of $D=4$ Reissner-Nordström black hole

Above we have demonstrated the existence of supersymmetric extremal $D=11$ and $D=10$ configurations with finite entropy which are built solely out of the fundamental p-branes of the corresponding theories (the 2-branes and the 5-branes of the M-theory and the 3-branes of type IIB theory) and reduce upon compactification to $D=4$ dyonic black hole backgrounds with regular horizon.

Namely, there exists an emdedding of a four dimensional dyonic black hole (in particular, of the non-dilatonic Reissner-Nordström black hole) into $D=11$ theory which corresponds to a combination of M-branes only. This may allow an application of the approach similar to the one of [14] to the derivation of the entropy (3.4) by counting the number of different BPS excitations of the $2\perp2\perp5\perp5$ M-brane configuration.

The $3\perp3\perp3\perp3$ configuration represents an embedding of the 1/8 supersymmetric dyonic $D=4$ black hole into type IIB superstring theory which is remarkable in that all four charges enter symmetrically. It is natural to expect that there should exist a microscopic counting of the BPS states which reproduces the Bekenstein-Hawking entropy in a (U-duality invariant) way that treats all four charges on an equal footing.

Although we hope to eventually attain a general understanding of this problem, in what follows we shall discuss the counting of BPS states for one specific example discussed above: the M-theory configuration (2.3),(2.2) of the three intersecting 5-branes with a common line. Even though the counting rules of M-theory are not entirely clear, we see an advantage to doing this from M-theory point of view as compared to previous discussions in the context of string theory [12,13]: the 11-dimensional problem is more symmetric. Furthermore, apart from the entropy problem, we may learn something about the M-theory.

[8] This is similar to what one finds for the 'unboosted' $5\perp5\perp5$ configuration (2.1),(2.2). As is well-known from 4-dimensional point of view, one does need *four* charges to get a regular behaviour of scalars near the horizon and finite area.

5.1. Charge quantization in M-theory and the Bekenstein-Hawking entropy

Upon dimensional reduction to four dimensions, the boosted $5\perp5\perp5$ solution (2.3), (2.2), reduces to the 4-dimensional black hole with three magnetic charges, P_1, P_2 and P_3, and an electric charge Q. The electric charge is proportional to the momentum along the intersection string of length L, $\mathcal{P} = 2\pi N/L$. The general relation between the coefficient Q in the harmonic function K appearing in (2.3) and the momentum along the $D = 5$ string (cf.(2.5)) wound around a compact dimension of length L is (see e.g. [27])

$$Q = \frac{2\kappa_{D-1}^2}{(D-4)\omega_{D-3}} \cdot \frac{2\pi N}{L} = \frac{\kappa_4^2 N}{L} = \frac{\kappa^2 N}{L^8} \ , \tag{5.1}$$

where $\kappa_4^2/8\pi$ and $\kappa^2/8\pi$ are the Newton's constants in 4 and 11 dimensions. All toroidal directions are assumed to have length L.

The three magnetic charges are proportional to the numbers n_1, n_2, n_3 of 5-branes in the (14567), (12367), and the (12345) planes, respectively (see (2.1),(2.3)). The complete symmetry between n_1, n_2 and n_3 is thus automatic in the 11-dimensional approach. The precise relation between P_i and n_i is found as follows. The charge q_5 of a $D = 11$ 5-brane which is spherically symmetric in transverse $d + 2 \leq 5$ dimensions is proportional to the coefficient P in the corresponding harmonic function. For $d + 2 = 3$ appropriate to the present case (two of five transverse directions are isotropic, or, equivalently, there is a periodic array of 5-branes in these compact directions) we get

$$q_5 = \frac{\omega_{d+1} d}{\sqrt{2}\kappa} P \ \rightarrow \ \frac{\omega_2 L^2}{\sqrt{2}\kappa} P = \frac{4\pi L^2}{\sqrt{2}\kappa} P \ . \tag{5.2}$$

At this point we need to know precisely how the 5-brane charge is quantized. This was discussed in [9], but we repeat the argument here for completeness. A different argument leading to equivalent results was presented earlier in [28]. Upon compactification on a circle of length L, the M-theory reduces to type IIA string theory where all charge quantization rules are known. We use the fact that double dimensional reduction turns a 2-brane into a fundamental string, and a 5-brane into a Dirichlet 4-brane. Hence, we have

$$T_2 \kappa^2 = T_1 \kappa_{10}^2 \ , \qquad\qquad T_5 \kappa^2 = T_4 \kappa_{10}^2 \ , \tag{5.3}$$

where the 10-dimensional gravitational constant is expressed in terms of the 11-dimensional one by $\kappa_{10}^2 = \kappa^2/L$. The charge densities are related to the tensions by

$$q_2 = \sqrt{2}\kappa T_2 \ , \qquad q_5 = \sqrt{2}\kappa T_5 \ , \tag{5.4}$$

and we assume that the minimal Dirac condition is satisfied, $q_2 q_5 = 2\pi$. These relations, together with the 10-dimensional expressions [29]

$$\kappa_{10} = g(\alpha')^2 \ , \qquad T_1 = \frac{1}{2\pi\alpha'} \ , \qquad \kappa_{10} T_4 = \frac{1}{2\sqrt{\pi\alpha'}} \ , \tag{5.5}$$

fix all the M-theory quantities in terms of α' and the string coupling constant, g. In particular, we find

$$\kappa^2 = \frac{g^3(\alpha')^{9/2}}{4\pi^{5/2}} \ , \qquad L = \frac{g\sqrt{\alpha'}}{4\pi^{5/2}} \ . \tag{5.6}$$

The tensions turn out to be

$$T_2 = \frac{2\pi^{3/2}}{g(\alpha')^{3/2}} \ , \qquad T_5 = \frac{2\pi^2}{g^2(\alpha')^3} \ . \tag{5.7}$$

Note that T_2 is identical to the tension of the Dirichlet 2-brane of type IIA theory, while T_5 – to the tension of the solitonic 5-brane. This provides a nice check on our results, since single dimensional reduction indeed turns the M-theory 2-brane into the Dirichlet 2-brane, and the M-theory 5-brane into the solitonic 5-brane. Note that the M-brane tensions satisfy the relation $2\pi T_5 = T_2^2$, which was first derived in [28] using toroidal compactification to type IIB theory in 9 dimensions. This serves as yet another consistency check.

It is convenient to express our results in pure M-theory terms. The charges are quantized according to[9]

$$q_2 = \sqrt{2}\kappa T_2 = n\sqrt{2}(2\kappa\pi^2)^{1/3} \ , \tag{5.8}$$

$$q_5 = \sqrt{2}\kappa T_5 = n\sqrt{2}(\frac{\pi}{2\kappa})^{1/3} \ , \tag{5.9}$$

i.e.

$$P_i = \frac{n_i}{2\pi L^2}(\frac{\pi\kappa^2}{2})^{1/3} \ . \tag{5.10}$$

The resulting expression for the Bekenstein-Hawking entropy of the extremal Reissner-Nordström type black hole, (2.6), (2.7), which is proportional to the area (2.4), is

$$S_{BH} = \frac{2\pi A_9}{\kappa^2} = \frac{8\pi^2 L^7}{\kappa^2}\sqrt{P_1P_2P_3Q} = 2\pi\sqrt{n_1n_2n_3N} \ . \tag{5.11}$$

This agrees with the expression found directly in $D=4$ [2,3,12,13].

In the case of the $2\perp 2\perp 5\perp 5$ configuration we find (for each pair of 2-brane and 5-brane charges) $q_2 = \frac{4\pi L^5}{\sqrt{2}\kappa}Q$, $q_5 = \frac{4\pi L^2}{\sqrt{2}\kappa}P$. The Dirac condition on unit charges translates into $q_2q_5 = 2\pi n_1 n_2$, where n_1 and n_2 are the numbers of 2- and 5-branes. We conclude that $Q_1P_1 = \frac{\kappa^2}{4\pi L^7}n_1n_2$. Then from (3.4) we learn that

$$S_{BH} = \frac{2\pi A_9}{\kappa^2} = \frac{8\pi^2 L^7}{\kappa^2}\sqrt{Q_1P_1Q_2P_2} = 2\pi\sqrt{n_1n_2n_3n_4} \ . \tag{5.12}$$

[9] In [30] it was argued that the 2-brane tension, T_2, satisfies $\kappa^2 T_2^3 = \pi^2/m_0$, where m_0 is a rational number that was left undetermined. The argument of [28], as well as our procedure [9], unambiguously fix $m_0 = 1/2$.

Remarkably, this result does not depend on the particular choice of M-brane quantization condition (choice of $m_0 = \pi^2\kappa^{-2}T_2^{-3}$) or use of D-brane tension expression since the $2\perp2\perp5\perp5$ configuration contains equal number of 2-branes and 5-branes. This provides a consistency check. Note also that the $D=4$ black holes obtained from the $2\perp2\perp5\perp5$ and from the $5\perp5\perp5$ M-theory configurations are not identical, but are related by U-duality. The equality of their entropies provides a check of the U-duality.

The same expression is obtained for the entropy of the $D=10$ configuration $3\perp3\perp3\perp3$ (4.3) (or related $D=4$ black hole). Each 3-brane charge q_3 is proportional to the corresponding coefficient Q in the harmonic function (cf. (5.2))

$$q_3 = \frac{1}{\sqrt{2}}(\frac{\omega_{d+1}d}{\sqrt{2}\kappa_{10}})Q \quad \to \quad \frac{\omega_2 L^3}{2\kappa_{10}}Q = \frac{2\pi L^3}{\kappa_{10}}Q \ , \tag{5.13}$$

where $\kappa_{10}^2/8\pi$ is the 10-dimensional Newton's constant and the overall factor $\frac{1}{\sqrt{2}}$ is due to the dyonic nature of the 3-brane. The charge quantization in the self-dual case implies (see [9]) $q_3 = n\sqrt{\pi}$ (the absence of standard $\sqrt{2}$ factor here effectively compensates for the 'dyonic' $\frac{1}{\sqrt{2}}$ factor in the expression for the charge).[10] Thus, $Q_i = \frac{\kappa_{10}}{2\sqrt{\pi}}n_i$, and the area (4.4) leads to the following entropy,

$$S_{BH} = \frac{2\pi A_8}{\kappa_{10}^2} = \frac{8\pi^2 L^6}{\kappa^2}\sqrt{Q_1Q_2Q_3Q_4} = 2\pi\sqrt{n_1n_2n_3n_4} \ . \tag{5.14}$$

5.2. Counting of the microscopic states

The presence of the factor $\sqrt{N}$ in S_{BH} (5.11) immediately suggests an interpretation in terms of the massless states on the string common to all three 5-branes. Indeed, it is well-known that, for a $1+1$ dimensional field theory with a central charge c, the entropy of left-moving states with momentum $2\pi N/L$ is, for sufficiently large N, given by[11]

$$S_{stat} = 2\pi\sqrt{\tfrac{1}{6}cN} \ . \tag{5.15}$$

We should find, therefore, that the central charge on the intersection string is, in the limit of large charges, equal to

$$c = 6n_1n_2n_3 \ . \tag{5.16}$$

[10] This agrees with the D3-brane tension, $\kappa_{10}T_3 = \sqrt{\pi}$, since in the self-dual case $q_p = \kappa_{10}T_p$.

[11] As pointed out in [31], this expression is reliable only if $N \gg c$. Requiring N to be much greater than $n_1n_2n_3$ is a highly asymmetric choice of charges. If, however, all charges are comparable and large, the entropy is dominated by the multiply wound 5-branes, which we discuss at the end of this section.

The fact that the central charge grows as $n_1 n_2 n_3$ suggests the following picture. 2-branes can end on 5-branes, so that the boundary looks like a closed string [20,32,33]. It is tempting to associate the massless states with those of 2-branes attached to 5-branes near the intersection point. Geometrically, we may have a two-brane with three holes, each of the holes attached to different 5-dimensional hyperplanes in which the 5-branes lie. Thus, for any three 5-branes that intersect along a line, we have a collapsed 2-brane that gives massless states in the $1+1$ dimensional theory describing the intersection. What is the central charge of these massless states? From the point of view of one of the 5-branes, the intersection is a long string in $5+1$ dimensions. Such a string has 4 bosonic massless modes corresponding to the transverse oscillations, and 4 fermionic superpartners. Thus, we believe that the central charge arising from the collapsed 2-brane with three boundaries is $4(1+\frac{1}{2})=6$.[12]

The upshot of this argument is that each triple intersection contributes 6 to the central charge. Since there are $n_1 n_2 n_3$ triple intersections, we find the total central charge $6n_1 n_2 n_3$. One may ask why there are no terms of order n_1^3, etc. This can be explained by the fact that all parallel 5-branes are displaced relative to each other, so that the 2-branes produce massless states only near the intersection points.

One notable feature of our argument is that the central charge grows as a product of three charges, while in all D-brane examples one found only a product of two charges. We believe that this is related to the peculiar n^3 growth of the near-extremal entropy of n coincident 5-branes found in [9] (for coincident D-branes the near-extremal entropy grows only as n^2). This is because the intersecting D-brane entropy comes from strings which can only connect objects pairwise. The 2-branes, however, can connect three different 5-branes. Based on our observations about entropy, we conjecture that the geometries where a 2-brane connects four or more 5-branes are forbidden (otherwise, for instance, the near-extremal entropy of n parallel 5-branes would grow faster than n^3). Perhaps such configurations are not supersymmetric and do not give rise to massless states.

The counting argument presented above applies to the configuration where there are n_1 parallel 5-branes in the (14567) hyperplane, n_2 parallel 5-branes in the (12367) hyperplane, and n_3 parallel 5-branes in the (12345) hyperplane. As explained in [31], if $n_1 \sim n_2 \sim n_3 \sim N$ we need to examine a different configuration where one replaces a number of disconnected branes by a single multiply wound brane. Let us consider, therefore, a single 5-brane in the (14567) hyperplane wound n_1 times around the y_1-circle, a single 5-brane in the (12367) hyperplane wound n_2 times around the y_1-circle, and a single 5-brane in

[12] Upon compactification on T^7, these massless modes are simply the small fluctuations of the long string in $4+1$ dimensions which is described by the classical solution (2.5). One should be able to confirm that the central charge on this string is equal to 6 by studying its low-energy modes.

the (12345) hyperplane wound n_3 times around the y_1-circle. Following the logic of [31], one can show that the intersection string effectively has winding number $n_1 n_2 n_3$: this is because the 2-brane which connects the three 5-branes needs to be transported $n_1 n_2 n_3$ times around the y_1-circle to come back to its original state.[13] Therefore, the massless fields produced by the 2-brane effectively live on a circle of length $n_1 n_2 n_3 L$. This implies [34] that the energy levels of the $1+1$ dimensional field theory are quantized in units of $2\pi/(n_1 n_2 n_3 L)$. In this theory there is only one species of the 2-brane connecting the three 5-branes; therefore, the central charge on the string is $c = 6$. The calculation of BPS entropy for a state with momentum $2\pi N/L$, as in [34,31], once again reproduces (5.11). While the end result has the form identical to that found for the disconnected 5-branes, the connected configuration is dominant when all four charges are of comparable magnitude [31]. Now the central charge is fixed, and the large entropy is due to the growing density of energy levels.

6. Black Hole Entropy in $D = 5$ and Discussion.

The counting arguments presented here are plausible, but clearly need to be put on a more solid footing. Indeed, it is not yet completely clear what rules apply to the 11-dimensional M-theory (although progress has been made in [14]). The rule associating massless states to collapsed 2-branes with three boundaries looks natural, and seems to reproduce the Bekenstein-Hawking entropy of extremal black holes in $D = 4$. Note also that a similar rule can be successfully applied to the case of the finite entropy $D = 5$ extremal dyonic black holes described in 11 dimensions by the 'boosted' $2 \perp 5$ configuration [15]. Another possible $D = 11$ embedding of the $D = 5$ black hole is provided by $2\perp 2\perp 2$ configuration [15]. The relevant $D = 10$ type IIB configuration is $3\perp 3$ (cf. (4.3)) with momentum flow along common string. In the case of $2 \perp 5$ configuration the massless degrees of freedom on the intersection string may be attributed to a collapsed 2-brane with a hole attached to the 5-brane and one point attached to the 2-brane. If the 5-brane is wound n_1 times and the 2-brane – n_2 times, the intersection is described by a $c = 6$

[13] The role of $n_1 n_2 n_3$ as the effective winding number is suggested also by comparison of the $D = 5$ solitonic string metric, (2.5), with the fundamental string metric, $ds^2 = V^{-1}(dudv + Kdu^2) + dx_s dx_s$, where the coefficient in the harmonic function V is proportional to the tension times the winding number of the source string (see e.g. [27]). After a conformal rescaling, (2.5) takes the fundamental string form with $V = H^3 = (F_1 F_2 F_3)^{-1}$ so that near $r = 0$ the $dudv$ part of it is multiplied by $P_1 P_2 P_3 \sim n_1 n_2 n_3$. Thus, the source string may be thought of as wound $n_1 n_2 n_3$ times around the circle.

theory on a circle of length $n_1 n_2 L$. Following the arguments of [31], we find that the entropy of a state with momentum $2\pi N/L$ along the intersection string is

$$S_{stat} = 2\pi\sqrt{n_1 n_2 N} \ . \tag{6.1}$$

This seems to supply a microscopic M-theory basis, somewhat different from that in [14], for the Bekenstein-Hawking entropy of $D=5$ extremal dyonic black holes.

We would now like to show that (6.1) is indeed equal to the expression for the Bekenstein-Hawking entropy for the 'boosted' $2\perp 5$ configuration [15] (cf. (5.11))

$$S_{BH} = \frac{2\pi A_9}{\kappa^2} = \frac{4\pi^3 L^6}{\kappa^2}\sqrt{QPQ'} \ . \tag{6.2}$$

Q and P are the parameters in the harmonic functions corresponding to the 2-brane and the 5-brane, and Q' is the parameter in the 'boost' function, i.e. $T^{-1} = 1 + Q/r^2$, $F^{-1} = 1 + P/r^2$, $K = 1 + Q'/r^2$. Note that here (cf. (5.1))

$$Q' = \frac{\kappa^2 N}{\pi L^7} \ , \qquad q_2 = \frac{4\pi^2 L^4}{\sqrt{2}\kappa} Q \ , \qquad q_5 = \frac{4\pi^2 L}{\sqrt{2}\kappa} P \ . \tag{6.3}$$

As in the case of the $2\perp2\perp5\perp5$ configuration, we can use the Dirac quantization condition, $q_2 q_5 = 2\pi n_1 n_2$, to conclude that $QP = \frac{\kappa^2}{4\pi^3 L^5} n_1 n_2$. This yields (6.1) when substituted into (6.2). A similar expression for the BPS entropy is found in the case of the completely symmetric $2\perp2\perp2$ configuration,

$$S_{BH} = \frac{4\pi^3 L^6}{\kappa^2}\sqrt{Q_1 Q_2 Q_3} = 2\pi\sqrt{n_1 n_2 n_3} \ , \tag{6.4}$$

where we have used the 2-brane charge quantization condition (5.8), which implies that $Q_i = n_i L^{-4} (\frac{\kappa}{\sqrt{2}\pi})^{4/3}$. Agreement of different expressions for the $D=5$ black hole entropy provides another check on the consistency of (5.8),(5.9).

Our arguments for counting the microscopic states applies only to the configurations where M-branes intersect over a string. It would be very interesting to see how approach analogous to the above might work when this is not the case. Indeed, black holes with finite horizon area in $D=4$ may also be obtained from the $2\perp2\perp5\perp5$ configuration in M-theory, and the $3\perp3\perp3\perp3$ one in type IIB, while in $D=5$ – from the $2\perp2\perp2$ configuration. Although from the $D=4,5$ dimensional point of view these cases are related by U-duality to the ones we considered, the counting of their states seems to be harder at the present level of understanding. We hope that a more general approach to the entropy problem, which covers all the solutions we discussed, can be found.

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